PHASE PORTRAIT CRITICAL POINTS OF THE SET OF ION DRIFT AND DIFFUSION EQUATIONS IN A DIELECTRIC LAYER UNDER CONCENTRATION COMPRESSION

Yuri B. Zolochevsky

124498, National Research University MIET, Moscow, Russia, e-mail: yzolochevsky@yahoo.com

Phase portrait critical points of the set of ion drift and diffusion equations in a dielectric layer under the effect of electric field and concentration compression at stationary state were investigated to a one-dimensional approximation. The case of concentration compression of a dielectric layer was considered as alinear term and a quadratic term of the elastic deformation power series extension by concentration compression ratio were kept.It was shown that under the negative value of the nonlinear diffusion coefficient and in the presence of homogeneously distributedimmovable negatively charged impurity in adielectric layer the set of the equationsmight haveoscillatingsolutions.

Ion concentration and electric field intensity were used as the phase plane coordinates. The flow at the phase plane wasparameterized by the value of the space coordinate in a dielectric layer.The analysis of solutions of theset of equations describing the behavior of mobile ions in a dielectric layer was reduced to the analysis of phase portraits of the system, while the integration of the system was reduced to finding the integral curves of the phase velocity vector field satisfying proper integral boundary conditions.

The structure of phase portraits of the system under study appeared to be reasonably complicated and the problem of phase curve plotting required an analytical analysis. Variety of range of values was considered for the

concentration ofimmovable negatively charged impurity in a dielectric layer, for linear and quadratic terms of the elastic deformation power series extension by concentration compression ratio as well as for the temperature. Parameter domains were studied for the set of equations with no critical points of phase space at all, with a saddle critical point as well as with a center critical point allowing oscillatory solutions.

Stability of the oscillatory solution in the neighborhood of the critical point was studied in regard to minor perturbations of the set of equations by the linearization of Taylor series in this area, by substitution of a definite form of the perturbation of the phase space arguments into the linearized system and by subsequent analysis of the derivable characteristic equation. It was established that oscillatory solutions were locally stable and under a minor perturbation the system transited once again into an oscillatory trajectory which parameters depended on the value of the perturbation.

It was shown that in the area of parameters where the system hada center critical point, the equations of the system had a Hamilton structure and consequently a function of ion concentration and electric field intensity existedwhich was invariant under the transformation of the space coordinate and represented the full energy of a unite of volume of dielectric. The invariance of this function under the transformation of the space coordinate corresponded to the invariance of the overall energy of the bulk density of the electric field and the bulk density of the electro-elastic interaction between ions and dielectric in an arbitrary point of a dielectric layer.

It was shown that the full energy of a unit of volume of dielectric and its first derivatives were continuous functions of the ion concentration and electric field intensity in a some neighborhoodof the critical point under study except for the point itself, its value was positive in this neighborhood except for the

point itself where its value wasequal to zero. The total derivative of the full energy of a unit of volume of dielectric was equal to zero in this neighborhood. Consequently the full energy of a unit of volume of dielectric might be taken as a Lyapunov function of the set of equations describing drift and diffusion in a dielectric layer under the effect of concentration compression in a neighborhood of a center critical point containing closed trajectories, and the critical point was globally stable.

The stability of the oscillating solutions in the nonstationary state was studied for the set of equations including both Poisson's equation and Fokker-Planck equation derived by substitution of the expression for the density of ion flow, under the negative value of the nonlinear diffusion coefficient and in the presence of homogeneously distributedimmovable negatively charged impurity, into the ion equation of continuity.

Self-similar solution of this system was studied by the phase trajectories method as well. Upon the set of equations was properly linearized and the form of phase arguments perturbation was chosen, interaction matrix components of the system were substituted into the characteristic equation and eigenvalues of the interaction matrix were established. Parameter domains were found with no critical points of phase space at all, with a saddle critical point, with a center critical point allowing oscillatory solutions as well as with a focus critical point.